CALCULATING CERTAIN QUANTITIES CHARACTERIZING AN ELEMENTARY EROSION EVENT DURING A SPARK DISCHARGE

N. V. Afanas'ev

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Formulas describing an individual electrode erosion event are derived. The relationship of this phenomenon to heat conduction and phase transitions at the electrode and to the electrical characteristics of the discharge are determined.

In our paper [1] we used concepts concerning the discrete emission of electrode material during a condensed spark discharge, and the hypothesis that this process is due to a surface heat source, to derive formulas for estimating some of the quantities which characterize an elementary act of emission of the vapor phase of the material. We now propose to refine the results of [1] by systematic consideration of the phase transitions.

The heat accumulated in some volume adjacent to the heel of the spark channel can be determined from the difference between the heat fluxes entering and emerging from this volume:

$$\Delta Q = \Delta Q_1 - \Delta Q_1' = \bar{q}\,\overline{\Delta s}\tau - \bar{q}'\,\overline{\Delta s}\tau. \tag{1}$$

Owing to the brevity of the discharge pulse, the second term of Eq. (1) can be assumed to be vanishingly small, so that

$$\Delta Q = \bar{q}\,\overline{\Delta s}\tau. \tag{2}$$

The heat ΔQ is expended in raising the temperature of this volume through heat conduction, and in phase transitions of the first kind,

$$\Delta Q = \lambda \frac{T_h - T_b}{l}\,\overline{\Delta s}\tau + r\Delta m_n. \tag{3}$$

The second term of Eq. (3) can be rewritten as

$$r\Delta m = r\gamma\Delta V = r\gamma\overline{\Delta s}l. \tag{4}$$

Substituting (4) into (3) and dividing (3) by $\Delta s\tau$, we obtain

$$\bar{q} = \frac{\Delta Q}{\overline{\Delta s}\tau} = \lambda\frac{T_h - T_b}{l} + \frac{r\gamma l}{\tau}. \tag{5}$$

Setting $l \sim (a\tau)^{1/2}$ and recalling that $a = \lambda/c\gamma$, we obtain

$$\tau = \frac{l^2 c\gamma}{\lambda}. \tag{6}$$

Substituting (6) into (5), we find that

$$\bar{q} = \lambda\frac{T_h - T_b}{l} + \frac{r\lambda}{cl} = \frac{\lambda}{l}\left(T_h - T_b + \frac{r}{c}\right). \tag{7}$$

Expression (7) implies that

$$l = \frac{\lambda}{\bar{q}}\left(T_h - T_b + \frac{r}{c}\right). \tag{8}$$

The above expression for l gives us new formulas for ΔV_n and Δm_n:

$$\Delta V_n = l\overline{\Delta s} = \frac{\lambda}{\bar{q}}\left(T_h - T_b + \frac{r}{c}\right)\overline{\Delta s}, \tag{9}$$

$$\Delta m_n = \Delta V\gamma = \frac{\lambda}{\bar{q}}\left(T_h - T_b + \frac{r}{c}\right)\gamma\overline{\Delta s}. \tag{10}$$

Similarly, τ, n, and m_n are given by

$$\tau = \frac{\lambda}{\bar{q}^2}\left(T_h - T_b + \frac{r}{c}\right)^2 c\gamma, \tag{11}$$

$$n = \frac{t}{\tau} = \frac{\bar{q}^2}{\lambda\left(T_h - T_b + \frac{r}{c}\right)^2 c\gamma}\,t, \tag{12}$$

$$m_n = \Delta m_n n = \frac{\bar{q}}{\left(T_h - T_b + \frac{r}{c}\right)c}\,\overline{\Delta s}t. \tag{13}$$

The right-hand sides of all the above formulas contain the quantity $\bar{q}$, which can be calculated by assuming that the energy liberated in the discharge gap is instantaneously dissipated through a spherical surface in the case of a point discharge, or through the side surface and bases of the cylindrical volume of the spark channel.

In the latter case the mean thermal flux density is given by

$$\bar{q} = k\frac{W}{st} = k\frac{CU^2}{\pi d(d + 2h)}. \tag{14}$$

Numerical calculation of the quantity $\bar{q}$ is quite difficult, since the coefficient k depends on the parameters of the discharge circuit and since h is a function of U and depends on the nature and state of the medium in which the discharge occurs.

In view of this it is expedient to express the $\bar{q}$ in Eqs. (8)–(13) in terms of electrical quantities characterizing the discharge pulse.

The energy liberated at the anode is approximated by the expression

$$W_a = (U_a + \varphi)\int_0^t i\,dt. \tag{15}$$

In the case of a condensed spark discharge, the electrical oscillations are damped, so that the instantaneous current is given by

$$i = i_0 e^{-\frac{R}{2L}t} \sin\omega t. \qquad (16)$$

The energy liberated in the time dt turns out to be

$$dW_a = (U_a + \varphi) i_0 e^{-\frac{R}{2L}t} \sin\omega t dt. \qquad (17)$$

The total energy liberated at the anode in the i-th half-period can be determined by integrating (17):

$$W_{ai} = (U_a + \varphi) i_0 \int_{\frac{T}{2}i}^{\frac{T}{2}(i+1)} e^{-\frac{R}{2L}t} \sin\frac{2\pi}{T} dt = -\frac{2LRT^2(U_0 + \varphi) i_0}{R^2T^2 + 16L^2\pi^2}\left[e^{-\frac{R}{2L}t}\left(\sin\frac{2\pi}{T}t + \frac{4\pi L}{RT}\cos\frac{2\pi}{T}t\right)\right]_{\frac{T}{2}i}^{\frac{T}{2}(i+1)} \qquad (18)$$

Expression (18) takes account of the decay of oscillations in the case of a condensed spark discharge and enables us to compute the energy, and therefore the current density at the anode, in any half-period of the discharge without additional assumptions concerning the values of k and h. The resulting formula is not a convenient one for practical calculations, however. It can be simplified by neglecting the variation of the period during the oscillations and by taking the values of i_0 directly from current oscillograms. This enables us to write the following expression for the energy liberated at the anode in the time dt:

$$dW_{ai} = (U_a + \varphi) i_{0i} \sin\omega t dt. \qquad (19)$$

The energy liberated in the i-th half-period can be found by integrating (19):

$$W_{ai} = (U_a + \varphi) i_{0i} \int_0^{T/2} \sin\omega t dt = \frac{(U_a + \varphi) i_{0i} T}{\pi}. \qquad (20)$$

Equation (20) allows us to compute the average densities of the thermal fluxes at the anode in any half-period of the discharge. The expression for $\bar{q}$ turns out to be

$$\bar{q}_{ai} = \frac{W_{ai}}{\overline{\Delta s}\frac{T}{2}} = \frac{2(U_a + \varphi) i_{0i}}{\overline{\Delta s}\pi}. \qquad (21)$$

Substituting (21) into (8)–(13), we obtain computation formulas for estimating l_i, ΔV_{ni}, Δm_{ni}, τ, n_i, and m_{ni} in the i-th half-period, expressed in terms of the electrical quantities characterizing the discharge process and the physical properties of the electrode material:

$$l_i = \frac{\lambda\overline{\Delta s}\pi}{2(U_a + \varphi) i_{0i}}\left(T_h - T_b + \frac{r}{c}\right), \qquad (22)$$

$$\Delta V_{ni} = \frac{\lambda(\Delta s)^2\pi}{2(U_a + \varphi) i_{0i}}\left(T_h - T_b + \frac{r}{c}\right), \qquad (23)$$

$$\Delta m_{ni} = \frac{\lambda(\overline{\Delta s})^2\pi\gamma}{2(U_a + \varphi) i_{0i}}\left(T_h - T_b + \frac{r}{c}\right), \qquad (24)$$

$$\tau_i = \frac{\lambda c\gamma(\overline{\Delta s})^2\pi^2}{[2(U_a + \varphi) i_{0i}]^2}\left(T_h - T_b + \frac{r}{c}\right)^2, \qquad (25)$$

$$n_i = \frac{[2(U_a + \varphi) i_{0i}]^2}{\lambda c\gamma\pi^2(\overline{\Delta s})^2\left(T_h - T_b + \frac{r}{c}\right)^2} t_i, \qquad (26)$$

$$m_{ni} = \frac{2(U_a + \varphi) i_{0i}}{\pi c\left(T_h - T_b + \frac{r}{c}\right)} t_i. \qquad (27)$$

The number of microcraters formed on the anode during the entire discharge pulse and the mass of matter emitted in the vapor phase from this electrode over the same period are

$$n_a = \sum_{i=1}^{k} \frac{[2(U_a + \varphi) i_{0i}]^2}{\lambda c\gamma\pi^2(\overline{\Delta s})^2\left(T_h - T_b + \frac{r}{c}\right)^2} t_i, \qquad (28)$$

$$m_{na} = \sum_{i=1}^{k} \frac{2(U_a + \varphi) i_{0i}}{\pi c\left(T_h - T_b + \frac{r}{c}\right)} t_i. \qquad (29)$$

Formulas (22)–(27) can be simplified in the case of rectangular discharge pulses. Here, the energy liberated at the anode can be written as

$$W_a = (U_a + \varphi) I \Delta t. \qquad (30)$$

The thermal flux density during the discharge pulse turns out to be

$$\bar{q}_a = \frac{(U_a + \varphi) I \Delta t}{\overline{\Delta s}\Delta t} = \frac{(U_a + \varphi) I}{\overline{\Delta s}}. \qquad (31)$$

Expressing I in terms of the current density at the anode, we obtain

$$\bar{q}_a = (U_a + \varphi) j. \qquad (32)$$

Substituting (31) and (32) into (8)–(13), we obtain a system of equations characterizing the processes at the anode in terms of the electrical parameters and physical properties of the electrode material in the case of rectangular pulse discharges:

$$l = \frac{\lambda\overline{\Delta s}\left(T_h - T_b + \frac{r}{c}\right)}{(U_a + \varphi) I} = \frac{\lambda\left(T_h - T_b + \frac{r}{c}\right)}{(U_a + \varphi) j}, \qquad (33)$$

$$\Delta V = \frac{\lambda(\overline{\Delta s})^2\left(T_h - T_b + \frac{r}{c}\right)}{(U_a + \varphi) I} = \frac{\lambda\left(T_h - T_b + \frac{r}{c}\right)\overline{\Delta s}}{(U_a + \varphi) j}, \qquad (34)$$

$$\Delta m = \frac{\lambda(\overline{\Delta s})^2 \gamma\left(T_h - T_b + \frac{r}{c}\right)}{(U_a + \varphi) I} = \frac{\lambda \overline{\Delta s}\, \gamma \left(T_h - T_b + \frac{r}{c}\right)}{(U_a + \varphi) j}, \tag{35}$$

$$\tau = \frac{\lambda c \gamma (\overline{\Delta s})^2 \left(T_h - T_b + \frac{r}{c}\right)^2}{[(U_a + \varphi) I]^2} = \frac{\lambda c \gamma \left(T_h - T_b + \frac{r}{c}\right)^2}{[(U_a + \varphi) j]^2}, \tag{36}$$

$$n = \frac{[(U_a + \varphi) I]^2}{\lambda c \gamma (\overline{\Delta s})^2 \left(T_h - T_b + \frac{r}{c}\right)^2} \Delta t = \frac{[(U_a + \varphi) j]^2}{\lambda c \gamma \left(T_h - T_b + \frac{r}{c}\right)^2} \Delta t, \tag{37}$$

$$m_n = \frac{(U_a + \varphi) I}{c \left(T_h - T_b + \frac{r}{c}\right)} \Delta t = \frac{(U_a + \varphi) j \overline{\Delta s}}{c \left(T_h - T_b + \frac{r}{c}\right)} \Delta t. \tag{38}$$

The equations for the cathode can be written out in similar form by taking the current and density values corresponding to the cathode and recalling that the thermal flux density at the cathode is given by

$$\bar{q}_c = \frac{U_c I}{\overline{\Delta s}}.$$

Equations (22)–(27) and (33)–(38) describe the elementary processes at the anode in the case of oscillatory discharging of the capacitance and for rectangular currents, respectively, in terms of the electrical quantities characterizing the discharge. They reflect the factors which affect the quantities occurring in the left-hand sides of the equations and enable us to estimate their values. Exact numerical computation of these quantities is still extremely difficult owing to the lack of reliable data concerning the values of the physical constants of the material at the high temperatures and pressures involved in spark discharges and concerning their functional dependences on these parameters.

NOTATION

l is the thickness of the layer of the quasi-vapor phase of the material emitted from the electrode surface as a result of an elementary explosion event; ΔV_n and Δm_n are the volume and mass of this layer; τ is the rise time of the elementary explosion; n is the number of elementary explosions: m_n is the mass of the vapor phase emitted from the electrode during a single discharge pulse; λ, γ, and c are the average thermal conductivity, density, and specific heat of the electrode material; $\bar{q}$ is the effective thermal flux density; T_h and T_b are the surface temperature of the heated electrode and the boiling point of the electrode material; a is the thermal diffusivity; $\overline{\Delta s}$ is the average area of the anode microcraters; t is the duration of the discharge pulse; r is the sum of the specific heats of melting and vaporization; k is a factor which represents the fraction of capacitor energy liberated in the discharge gap; C is the capacitor capacitance; U is the voltage across the capacitor plates; d and h are the diameter and length of the spark channel; i_{0i} is the peak value of the current in the i-th half-period; e is the base of natural logarithms; R and L are the resistance and inductance of the discharge circuit; U_a and U_c are the anode and cathode potential drops; φ is the electron work function; T is the period; k is the number of half-periods.

REFERENCES

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Polytechnic Institute, Minsk